NOTE ON PERSPECTIVE PROJECTION.

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In order to give verisimilitude to the picture the centre of projection must be at considerable distance from the original and the picture plane; this gives rise to the necessity for drawing long lines through distant points by the use of either the plans of the rays directly, or the employment of *diagonals* and their vanishing point. In the latter case, the difficulty is somewhat alleviated by means of *reduced distances;* in other words, by using *diagonals* which make a large angle with the picture plane, and whose vanishing point is correspondingly near the centre of the picture. The objection to this last method is that in order to locate the trace of the diagonal with the perspective plane a subdivision of the distance of the point in space from the picture plane must be used, which necessitates an additional construction for each point, involving time and a certain amount of inaccurary due to transferring distances. Where the plans of the rays concurring in the distant plan of the centre of projection are used, the centro-linead is inaccurate, and the use of a harmonic range with the quadrilateral construction too tedious.

It is believed that the following construction obviates some of the difficulties met with in drawing such projections by the methods now in use.

Let the orthogonal plan of the original lie in any horizontal plane α; select any straight line in this plane as the trace with it of the vertical picture plane β. Let the centre of projection be any point in space, and from it project the orthogonal plan upon the plane β, into what will be called the perspective plan. Rebat β into α. The centre of projection comes down into α by describing a quadrant of a circle whose plane is normal to α and β, and whose centre is the orthogonal plan of the centre of projection. The plan and its perspective are now superposed into two homological figures in a plane, the trace of α with β being the axis, and the development of the centre of projection the centre of homology. If the perspective of any one point of the plan be known, the whole perspective plan can at once be drawn (Cremona, Proj. Geom., Art. 22, *et seq.*).

It is obvious that the position of the centre of projection may always be selected, without loss of advantageous position, so that its development into α may have any position we choose in that plane with respect to the plan. The construction of the perspective plan may thus be confined to the desired area of drawing surface.

In particular, let the centre of projection be equidistant from α and β, in front of β and above α, the orthogonal plan being also in front of β. After the

rebatment of β back into α the centre is found at O, the intersection of the axis with the plan r of the central ray, and the perspective of any point P on r comes up into α on r to a point P' homological with P, the distance OP is a mean proportional between PP' and PC', where C' is the orthogonal plan of the centre of projection. In other words, if P be selected so that its distance from O is one nth of its distance from C', then PP' is one nth of OP. This fixes a pair of homological points necessary to complete the construction of the perspective plan.

It is well to let the orthogonal plan touch the axis; this condenses the drawing, and literally superposes the perspective plan on the orthogonal; each perspective point is near its original and easily identified.

Points in the picture lie on normals to the axis through points of the perspective plan, where they are located by means of the perspective of *perpendiculars* which vanish in the centre of the picture, and pass through their traces with β. These traces are referred to the *prime vertical* and the *horizon* as axes. They lie on normals to the axis of homology through points in the orthogonal plan, where they are located by the distance of the original point from the plane of the horizon, as determined by a given orthogonal elevation, or equivalent. These latter are the only measures taken, or which have to be transferred, with dividers. This method gives oblique intersections near the vertical through the centre of the picture. The diagonal may be used to good advantage in that case, the plan of any ray serving as the direction for a diagonal which locates the vanishing point; otherwise the vanishing point may be assumed at pleasure and the plan of the ray passing through it constructed as above, and perspectives of particular diagonals constructed by the usual methods.